Student Number:
Set:

SHORE

2008

Trial HSC Examination

Mathematics

General Instructions

- Reading time − 5 minutes
- Working time − 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided on the back page of this question paper
- All necessary working should be shown in every question

Total marks − 120

- Attempt Questions 1 − 10
- All questions are of equal value
- Start each question in a new writing booklet
- Write your examination number on the front cover of each booklet to be handed in
- If you do not attempt a question, submit a blank booklet marked with your examination number and “N/A” on the front cover

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Total Marks − 120
Attempt Questions 1−10
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Evaluate $\sqrt{9 + e^2}$ correct to three significant figures. **2**

(b) Solve $6 - 2x < 14$. **2**

(c) If $\dfrac{4}{2 + \sqrt{3}} = a + b\sqrt{3}$, find the values of a and b. **2**

(d) Find the sum of the first ten terms of the arithmetic series $4\frac{1}{2} + 3 + 1\frac{1}{2} + \ldots..$ **2**

(e) Factorise $2a^2 + 6ay - a - 3y$. **2**

(f) Find a primitive of $\dfrac{2}{x} - \dfrac{1}{x^2}$. **2**

Question 2 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Differentiate each of the following with respect to x:

(i) $\dfrac{e^x}{1+x}$ **2**

(ii) $\cos^2 3x$ **2**

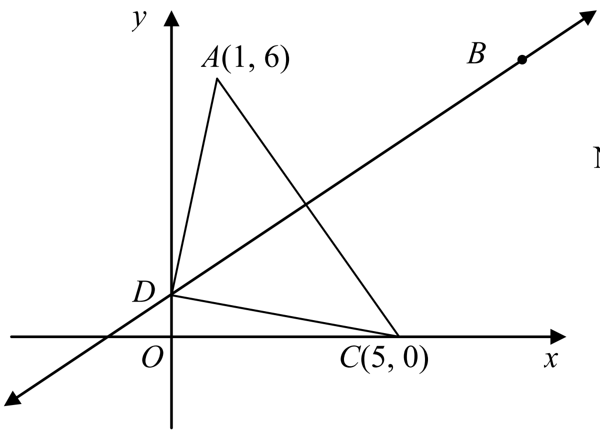
(b) Find $\int (1-\sin \pi x)\, dx$. **2**

(c) Evaluate $\displaystyle\int_1^{e^2} \frac{dx}{2x}$. Write your answer in simplest form. **3**

(d) Find the equation of the tangent to the curve $y = x\sin 2x$ at the point on the curve where $x = \dfrac{3\pi}{2}$. **3**

Question 3 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



NOT TO SCALE

In the diagram above, the coordinates of A and C are $(1, 6)$ and $(5, 0)$ respectively. The line BD has equation $2x - 3y + 3 = 0$ and meets the y-axis in D.

(i) The point M is the midpoint of AC. Show that M has coordinates $(3, 3)$. **1**

(ii) Show that M lies on BD. **1**

(iii) Find the gradient of the line AC. **1**

(iv) Show that BD is perpendicular to AC. **2**

(v) Explain why the quadrilateral $ABCD$ is a kite regardless of the position of B. **1**

Question 3 continues

(b) Consider the quadratic equation $x^2 - kx + (2k-1)^2 = 0$. **2**
Find the value(s) of k if the sum of the roots is equal to the product of the roots.

(c) Alex is training for a local marathon. He has trained by completing practice runs over the marathon course. So far he has completed three practice runs with times shown below.

Week 1	3 hours
Week 2	2 hours 51 minutes
Week 3	2 hours 42 minutes 27 seconds

(i) Show that these times form a geometric series with a common ratio $r = 0.95$. **1**

(ii) If this series continues, what would be his expected time in Week 5? Write your answer correct to the nearest second. **1**

(iii) The previous winning time for the marathon was 2 hours and 6 minutes. For how many weeks must Alex keep practising to be able to run the marathon in less than the previous winning time? **2**

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet **Marks**

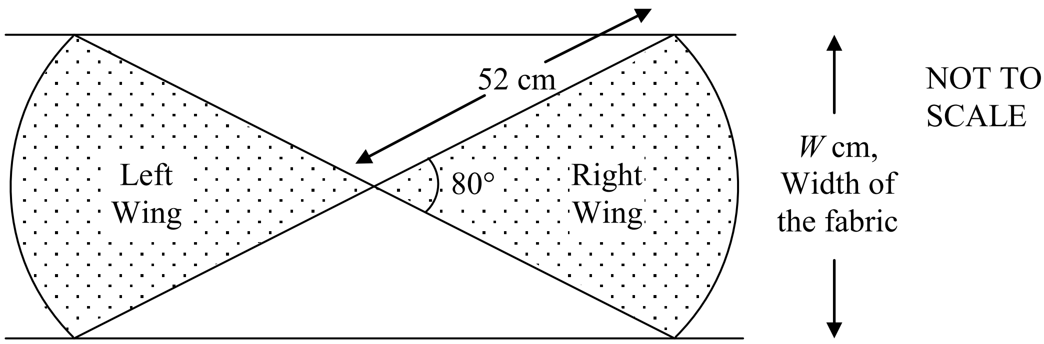
(a) Show that $\sqrt{\dfrac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = \tan x$. **2**

(b) Two dice are painted so that the first die has four blue and two red faces and the second die has one blue and five red faces. The two dice are rolled simultaneously.

(i) What is the probability that different colours show on the uppermost faces of the dice? **2**

(ii) What is the probability that at least one die shows blue on its uppermost face? **2**

(c)



Tim is making a pair of fairy wings for his little sister's ballet concert, i.e. one left and one right wing. Each wing is in the shape of a sector of a circle. The angle at the centre of the sector is 80°. The above diagram shows the placement of the two wings on a piece of fabric with width, W centimetres.

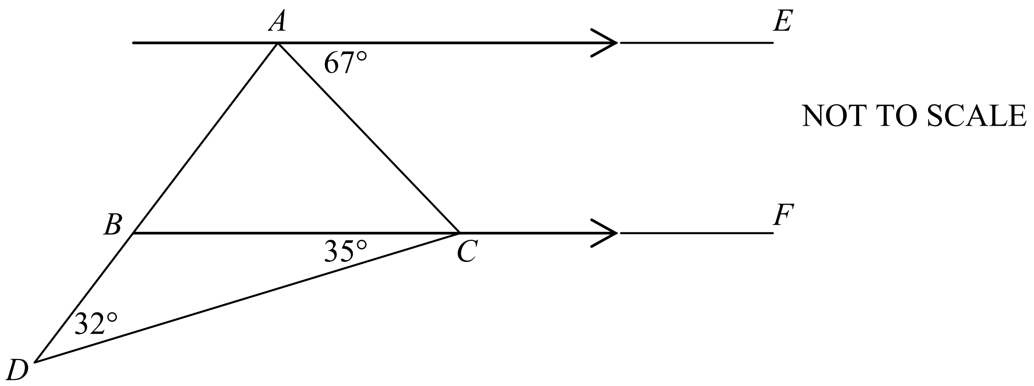
(i) The radius of each sector is 52 cm. Calculate the value of W. Give your answer correct to the nearest centimetre. **2**

(ii) Calculate the area covered by the set of two wings. Give your answer correct to the nearest square centimetre. **2**

(iii) The set of wings has a trim along each radius and each arc. Calculate the total length of trim required for the set of wings. Give your answer correct to the nearest centimetre. **2**

Question 5 (12 marks) Use a SEPARATE writing booklet **Marks**

(a)



In the diagram above AE is parallel to BF, C lies on BF, and AB is produced to D such that $\angle ADC = 32°$, $\angle BCD = 35°$, $\angle EAC = 67°$.

Copy or trace this diagram into your writing booklet.

By considering the size of angles, show that ΔABC is isosceles. **2**

(b) A particle is moving on the x-axis and is initially at the origin. Its velocity, v metres per second, at time t seconds is given by

$$v = \frac{4}{t+1} - 2t .$$

(i) What is the initial velocity of the particle? **1**

(ii) Find the time when the particle changes direction. **2**

(iii) Find the acceleration of the particle when $t = 3$. **2**

(iv) Find the distance travelled by the particle in the third second. Write your answer correct to 2 decimal places. **3**

(c) Show that the quadratic equation $mx^2 + (m-4)x - 4 = 0$ has real roots for all values of m. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) For the function $y = \dfrac{x^4 - 4x^3}{4}$

(i) Find the coordinates of the points where the curve crosses the axes. **2**

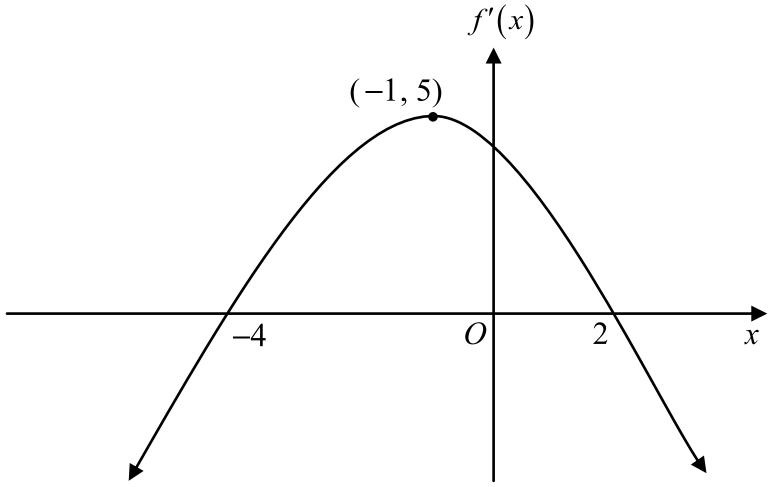
(ii) Find the coordinates of the stationary points and determine their nature. **3**

(iii) Find the coordinates of any points of inflexion. **2**

(iv) Sketch the graph of $y = \dfrac{x^4 - 4x^3}{4}$ indicating clearly the intercepts, stationary points and points of inflexion. **2**

(v) For what values of x is the curve concave up? **1**

(b) For a certain function $y = f(x)$, the sketch of $y = f'(x)$ is shown.



$y = f'(x)$ has a turning point at $(-1, 5)$ and cuts the x-axis at -4 and 2. **2**
Give the x coordinates of the stationary points on $y = f(x)$ and indicate if they are maxima or minima.

Question 7 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Consider the parabola given by $x^2 + 8y + 16 = 0$.

(i) Find the coordinates of the vertex. **2**

(ii) Find the coordinates of the focus. **1**

(iii) State the equation of the directrix. **1**

(b) The populations of two towns G and H are given by the exponential growth models

$$P_G = 20\,000e^{0.0318t} \quad \text{for town } G$$

$$P_H = 14\,000e^{kt} \quad \text{for town } H$$

where k is a constant, and t is the time in years since January 1st 2000.

It is known on January 1st 2000, the populations were 20 000 for town G and 14 000 for town H. By January 1st 2003 the population of town G had grown to 22 000 and the population of town H had grown to 18 500.

(i) Find the value of k correct to 4 decimal places. **2**

(ii) After how many years will the population of town H become greater than the population of town G? **2**

(c) (i) Sketch $y = 1 + \sin 2x$ for $0 \le x \le \pi$, showing all essential features. **2**

(ii) Find the values of x where the graph of $y = 1 + \sin 2x$ intersects with $y = 1\frac{1}{2}$ for $0 \le x \le \pi$. **2**

Question 8 (12 marks) Use a SEPARATE writing booklet **Marks**

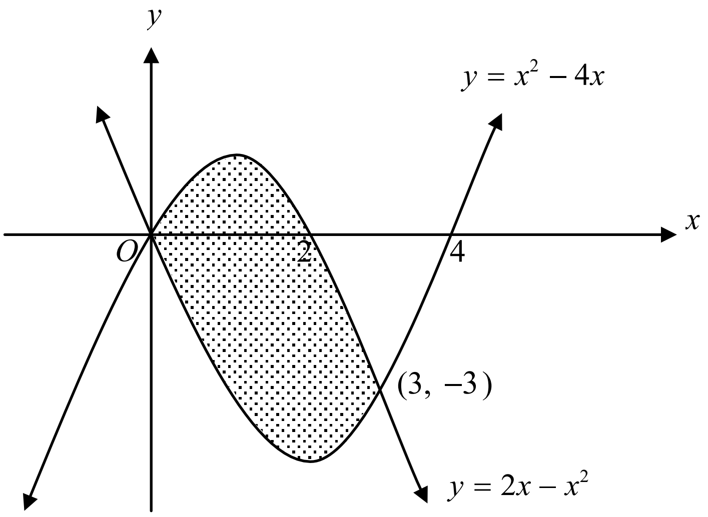
(a) Evaluate $\sum_{n=2}^{5} n^2 - 1$. **1**

(b) For what values of x will the following geometric series have a limiting sum? **2**

$$1 + (3 - x) + (3 - x)^2 + \text{.........}$$

(c) The diagram shows the two curves $y = x^2 - 4x$ and $y = 2x - x^2$. **3**
The two curves intersect at the points (0, 0) and $(3, -3)$.

Find the size of the area enclosed between the two curves.



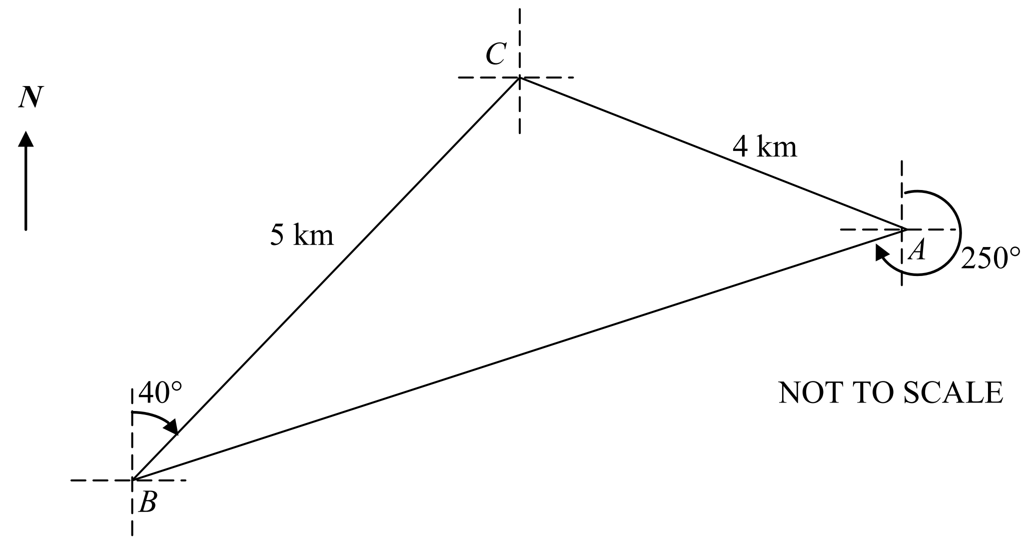
NOT TO SCALE

(d) Find an approximation for $\int_1^3 g(x)\,dx$ by using Simpson's Rule with the values in the table below. **2**

x	1	1.5	2	2.5	3
$g(x)$	12	8	1.5	3	5

Question 8 continues

(e)



$A, B,$ and C are markers in an orienteering course. $AC = 4$ km and $BC = 5$ km.
The bearing of C from B is 040°T. The bearing of B from A is 250°T.

Copy or trace this diagram into your writing booklet.

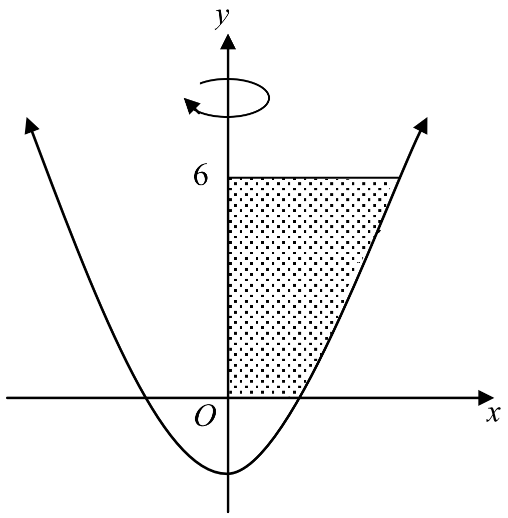
(i) Find the size of $\angle CAB$ correct to the nearest degree. **3**

(ii) Hence find the bearing of A from C. **1**

End of Question 8

Question 9 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) **3**



NOT TO SCALE

The diagram shows the region bounded by the curve $y = 2x^2 - 2$, the line $y = 6$, and the x and y axes.

Find the exact volume of the solid of revolution formed when the shaded region is rotated about the y axis.

(b) Angus plays computer games competitively. From past experience, Angus has a 0.8 chance of winning a game of *Beastie* and a 0.6 chance of winning a game of *Dragonfire*. In one afternoon of competition he plays two games of *Beastie* and one of *Dragonfire*.

(i) What is the probability that he will win all three games? **1**

(ii) What is the probability that he wins at least one game of *Beastie*? **2**

Question 9 continues

(c) A car dealership has a car for sale for a cash price of \$20 000. It can also be bought on terms over three years. The first six months are interest free and after that interest is charged at the rate of 1% per month on the balance owing for that month. Repayments are to be made in equal monthly instalments of \$$M$ with the first repayment applied at the end of the first month.

A customer agrees to buy the car on these terms.

Let \$$A_n$ be the amount owing at the end of the nth month.

(i) Find an expression for A_6. **1**

(ii) Show that $A_8 = (20\ 000 - 6M)1.01^2 - M(1 + 1.01)$. **1**

(iii) Find an expression for A_{36}. **2**

(iv) Find the value of M. **2**

End of Question 9

Question 10 (12 marks) Use a SEPARATE writing booklet **Marks**

(a) A plant nursery has a watering system which repeatedly fills a storage tank before emptying its contents to water different sections of the nursery.
The volume, V cubic metres, of water in the tank at time t minutes is given by the equation

$$V = 2 - \sqrt{3}\cos t - \sin t.$$

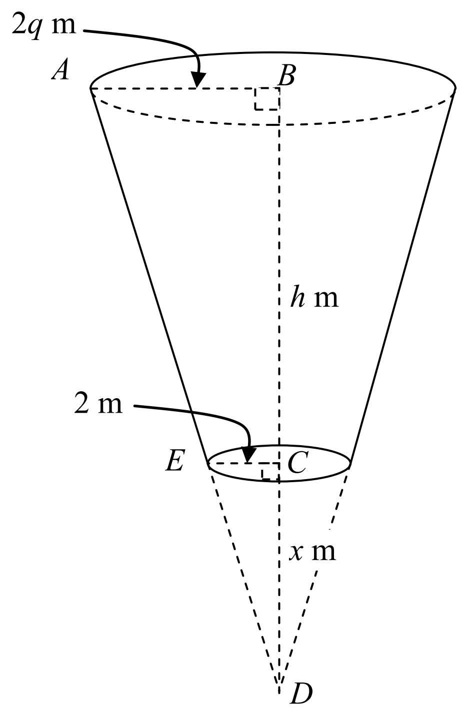
(i) Give an equation for $\dfrac{dV}{dt}$, the rate of change of the volume after t minutes. **1**

(ii) Is the tank initially filling or emptying? Give a reason for your answer. **1**

(iii) At what time does the tank first become completely full and what is its capacity when full? **3**

Question 10 continues

(b) A truncated cone is to be used as a part of a hopper for a grain harvester. It has a total height of h metres. The upper radius is to be q times greater than the lower radius which is 2 metres.



$AB = 2q$ metres
$BC = h$ metres
$EC = 2$ metres
$CD = x$ metres

NOT TO SCALE

(i) Given that triangles ABD and ECD are similar, show that the height, x metres, of the removed section of the original cone is given by **2**

$$x = \frac{h}{q-1}.$$

(ii) Show that the volume of the truncated cone is given by **2**

$$V = \left(\frac{4\pi h}{3}\right)\left(q^2 + q + 1\right).$$

(iii) The sum of the upper radius, the lower radius and the height of the truncated cone must total 12 metres. Calculate the value of q if the hopper is to have a maximum volume. You must provide a reason why your calculation results in a maximum volume. **3**

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \ , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

2U TRIAL HSC SOLUTIONS – 2008

Question 1

(a) $\sqrt{9+e^2} = 4.0483\ldots$

$= \underline{\underline{4.05}}$ to 3 s.f.

[2]

(b) $6 - 2x < 14$

$-2x < 8$

$\underline{\underline{x > -4}}$ [2]

(c) $\dfrac{4}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{4(2-\sqrt{3})}{4-3}$

$= 8 - 4\sqrt{3}$

$= a + b\sqrt{3}$

$\therefore \underline{\underline{a = 8}},\ \underline{\underline{b = -4}}$ [2]

(d) $a = 4\frac{1}{2}$ $d = -1\frac{1}{2}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}\left[2(4\tfrac{1}{2}) + 9(-1\tfrac{1}{2})\right]$

$= 5\left[9 - 13\tfrac{1}{2}\right]$

$= \underline{\underline{-22\tfrac{1}{2}}}$ [2]

(e) $2a^2 + 6ay - a - 3y$

$= 2a(a+3y) - 1(a+3y)$

$= \underline{\underline{(2a-1)(a+3y)}}$ [2]

(f) $\dfrac{dy}{dx} = \dfrac{2}{x} - \dfrac{1}{x^2}$

not needed

$\underline{\underline{y = 2\ln x + \frac{1}{x}}} + C$

[2]

Question 2

(a) (i) $y = \dfrac{e^x}{1+x}$

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{(1+x)e^x - e^x(1)}{(1+x)^2}$$

$$= \frac{e^x + xe^x - e^x}{(1+x)^2}$$

$$= \frac{xe^x}{(1+x)^2}$$

[2]

(ii) $y = (\cos 3x)^2$

$$y' = 2(\cos 3x) \times -3\sin 3x$$

$$= -6\cos 3x \sin 3x$$

[2]

(b) $\int (1 - \sin \pi x)\, dx$

$$= x + \frac{1}{\pi}\cos \pi x + C$$

[2]

(c) $\int_1^{e^2} \dfrac{dx}{2x}$

$$= \frac{1}{2}\int_1^{e^2} \frac{1}{x}\, dx$$

$$= \frac{1}{2}\Big[\ln x\Big]_1^{e^2}$$

$$= \frac{1}{2}\left(\ln e^2 - \ln 1\right]$$

$$= \frac{1}{2}\left[2 - 0\right]$$

$$= 1$$

[3]

(d) $y = x \sin 2x$

$$y' = uv' + vu'$$

$$= x \times 2\cos 2x + \sin 2x \times 1$$

$$= 2x\cos 2x + \sin 2x$$

at $x = \frac{3\pi}{2}$ $\quad m_T = 2\left(\frac{3\pi}{2}\right)\cos 2\left(\frac{3\pi}{2}\right) + \sin 2\left(\frac{3\pi}{2}\right)$

$$= 3\pi \times -1 + 0$$

$$= -3\pi$$

$$y = \frac{3\pi}{2}\sin 2\left(\frac{3\pi}{2}\right)$$

$$= 0$$

Eq'n of tangent is

$$y - 0 = -3\pi\left(x - \frac{3\pi}{2}\right)$$

$$y = -3\pi x + \frac{9\pi^2}{2}$$

[3]

Question 3

(a) (i) $M = \left(\frac{1+5}{2}, \frac{6+0}{2}\right)$

$= \underline{\underline{(3, 3)}}$ [1]

(ii) Sub in $(3,3)$

$LHS = 2x - 3y + 3$

$= 2(3) - 3(3) + 3$

$= 6 - 9 + 3$

$= 0$

$\underline{\underline{= RHS}}$ [1]

(iii) $m_{AC} = \frac{6-0}{1-5}$

$= \underline{\underline{\frac{-3}{2}}}$ [1]

(iv) $BD \rightarrow 2x - 3y + 3 = 0$

i.e. $y = \frac{2}{3}x + 1$

$\therefore m_{BD} = \frac{2}{3}$

$m_{BD} \times m_{AC} = \frac{2}{3} \times -\frac{3}{2}$

$= \underline{-1}$

$\therefore \underline{\underline{BD \perp AC}}$ [2]

(v) One diagonal bisects the other diagonal at right angles $\therefore$ a kite [1]

(b) $\alpha + \beta = \alpha\beta$

$\frac{-b}{a} = \frac{c}{a}$

$-b = c$

$\therefore k = (2k-1)^2$

$k = 4k^2 - 4k + 1$

$0 = 4k^2 - 5k + 1$

$0 = (4k-1)(k-1)$

$\underline{\underline{k = \frac{1}{4}}}$ or $\underline{\underline{k = 1}}$ [2]

(c) (i) $\frac{2h\ 42m\ 27s}{2h\ 51m} = 0{\cdot}95$

$\frac{2h\ 51m}{3h} = 0{\cdot}95$ [1]

$\therefore \underline{\underline{r = 0{\cdot}95}}$

(ii) Time in W5 $= 3h \times (0{\cdot}95)^4$

$= \underline{\underline{2h\ 26m\ 37s}}$

[1]

(iii) Time in W7 $= 3h \times (0{\cdot}95)^6$

$= \underline{2h\ 12m\ 19s}$

Time in W8 $= 3h \times (0{\cdot}95)^7$

$= \underline{2h\ 5m\ 42s}$

$\therefore$ He will take $\underline{8 \text{ weeks}}$ to better previous best time.

[2]

Question 4

(a) $LHS = \sqrt{\dfrac{(\operatorname{cosec}^2 x - \cot^2 x) - \cos^2 x}{\cos^2 x}}$

$= \sqrt{\dfrac{1 - \cos^2 x}{\cos^2 x}}$

$= \sqrt{\dfrac{\sin^2 x}{\cos^2 x}}$

$= \dfrac{\sin x}{\cos x}$

$= \tan x$

$= RHS$ [2]

(b) 4B 2R   1B 5R

(i) $P(BR \text{ or } RB)$

$= \frac{4}{6} \times \frac{5}{6} + \frac{2}{6} \times \frac{1}{6}$

$= \frac{11}{18}$ [2]

(ii) $P(BR \text{ or } RB \text{ or } BB)$

$= \frac{11}{18} + \frac{4}{6} \times \frac{1}{6}$

$= \frac{13}{18}$ [2]

[OR $P(\text{at least 1 Blue})$

$= 1 - P(RR)$

$= 1 - \frac{2}{6} \times \frac{5}{6}$

$= \frac{13}{18}$]

(c) (i) By cosine rule

$w^2 = 52^2 + 52^2 - 2 \times 52 \times 52 \times \cos 80^\circ$

$w^2 = 4468.91\ldots$

$w = 66.849\ldots$

$= 67 \text{ cm}$ (nearest cm) [2]

(ii) $A = \frac{1}{2} r^2 \theta \times 2$

$= \frac{1}{2} \times 52^2 \times \frac{80\pi}{180} \times 2$

$= 3775.496\ldots$

$= 3775 \text{ cm}^2$ (nearest cm²) [2]

(iii) $P = 2 \times r\theta + 4 \times 52$

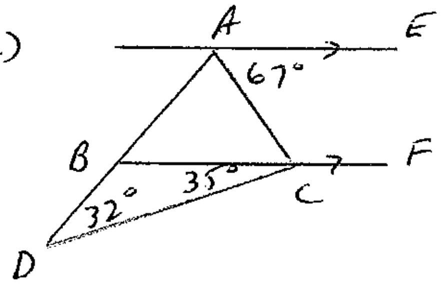
$= 2 \times 52 \times \frac{80\pi}{180} + 208$

$= 353.21\ldots$

$= 353 \text{ cm}$ (nearest cm) [2]

Question 5

(a)



$\angle ACB = 67°$ alt $\angle$'s, $AE \parallel BF$

$\angle ABC = 67°$ ext $\angle$ of $\triangle ABC$ equals sum of 2 opp. int. $\angle$'s

$\therefore$ $\triangle ABC$ is isos. base $\angle$'s equal

[2]

(b) $v = \dfrac{4}{t+1} - 2t$

(i) when $t = 0$, $v = \dfrac{4}{0+1} - 2(0)$

$= 4\ m/s$ [1]

(ii) when $v = 0$, $0 = \dfrac{4}{t+1} - 2t$

$0 = 4 - 2t(t+1)$

$0 = 4 - 2t^2 - 2t$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = -2 \quad t = 1$

$\therefore$ particle changes direction after 1 second [2]

(b) (iii) $v = \dfrac{4}{t+1} - 2t$

$a = \dfrac{-4}{(t+1)^2} - 2$

when $t = 3$, $a = \dfrac{-4}{(3+1)^2} - 2$

$= -\dfrac{4}{16} - 2$

$= -2\frac{1}{4}\ m/s^2$ [2]

(iv) $x = \displaystyle\int_2^3 \left(\frac{4}{t+1} - 2t\right) dt$

$= \left[4\ln(t+1) - t^2\right]_2^3$

$= (4\ln 4 - 9) - (4\ln 3 - 4)$

$= 4\ln 4 - 4\ln 3 - 5$

$= -3.85$

$\therefore$ distance travelled in third second was $3.85\ m$ [3]

(c) For real roots $\Delta \geq 0$

$\Delta = b^2 - 4ac$

$= (m-4)^2 - 4 \times m \times -4$

$= m^2 - 8m + 16 + 16m$

$= m^2 + 8m + 16$

$= (m+4)^2$

≥ 0 for all values of m

[2]

Question 6

(a) (i) $y = \dfrac{x^4 - 4x^3}{4}$

x intercepts: $0 = \dfrac{x^4 - 4x^3}{4}$

$0 = x^3(x-4)$

$x = 0 \quad x = 4$

$\therefore$ intercepts are $(0,0)$ $(4,0)$

[2]

(ii) $y = \dfrac{x^4 - 4x^3}{4}$

$y' = \dfrac{4x^3 - 12x^2}{4}$

$= x^3 - 3x^2$

$y'' = 3x^2 - 6x$

For S.P. $x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0 \quad x = 3$

at $x=0$ $\quad y = 0 \quad y'' = 0$

at $x = 3$ $\quad y = \dfrac{81 - 4(27)}{4} = -6\frac{3}{4}$ $\quad y'' = 3(3)^2 - 6(3) = 9 (>0)$

$\therefore (0,0)$ is H.P.O.I

$(3, -6\frac{3}{4})$ is a minimum T.P.

[3]

(a) (iii) For P.O.I.

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$

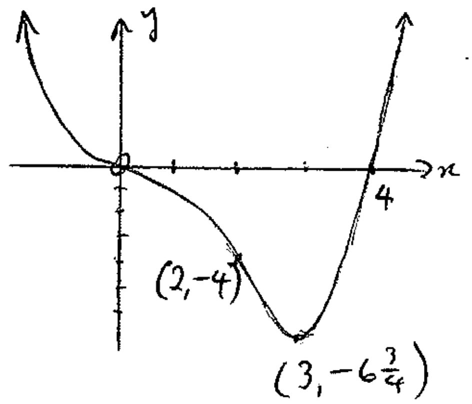
at $x=0$ H.P.O.I. at $(0,0)$ by above

at $x=2$ $\quad y = \dfrac{16-32}{4} = -4$

x	2^-	2	2^+
y''	$-$	0	$+$
	∩	.	∪

$\therefore$ P.O.I. at $(2, -4)$ [2]

(iv)



[2]

(v) Concave up for $x < 0$, $x > 2$ [1]

(b) S.P's when $f'(x) = 0$

$\therefore$ S.P's at $x = -4$ minimum $(f''(x) > 0)$

at $x = 2$ maximum $(f''(x) < 0)$

[2]

Question 7

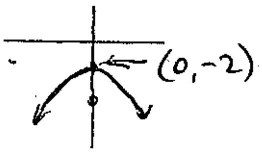
(a) (i) $x^2 + 8y + 16 = 0$

$$x^2 = -8y - 16$$
$$x^2 = -8(y+2)$$

vertex $(0,-2)$ [2]

(ii) $4a = 8$

$a = 2$



focus $(0,-4)$ [1]

(iii) directrix $y = 0$ (x-axis)

[1]

(b) (i) $18\,500 = 14\,000\,e^{3k}$

$$\frac{185}{140} = e^{3k}$$
$$3k = \ln\left(\frac{185}{140}\right)$$
$$k = \frac{1}{3}\ln\left(\frac{37}{28}\right)$$
$$= 0.0929 \quad (4\text{d.p})$$

[2]

(ii) $14\,000\,e^{0.0929t} > 20\,000\,e^{0.0318t}$

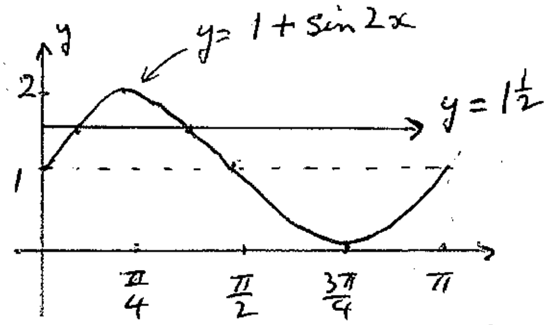
$$\frac{14}{20} > \frac{e^{0.0318t}}{e^{0.0929t}}$$
$$0.7 > e^{-0.0611t}$$
$$\ln 0.7 > -0.0611t$$
$$\frac{\ln 0.7}{-0.0611} < t$$
$$t > 5.837\ldots$$

[2]

$\therefore P_H > P_G$ after 6 years

(can be done by guess + check)

(c) (i)



[2]

(ii) $1 + \sin 2x = 1\frac{1}{2}$

$$\sin 2x = \frac{1}{2}$$
$$2x = \frac{\pi}{6}, \frac{5\pi}{6}$$
$$x = \frac{\pi}{12}, \frac{5\pi}{12}$$

[2]

Question 8

(a) $\sum_{n=2}^{5} n^2 - 1$

$= 3 + 8 + 15 + 24$

$= \underline{\underline{50}}$ [1]

(b) For S_∞ $\quad -1 < 3 - x < 1$

$-4 < -x < -2$

$4 > x > 2$

[2] $\quad \underline{\underline{2 < x < 4}}$

(c) $A = \int_0^3 (2x - x^2) - (x^2 - 4x)\, dx$

$= \int_0^3 6x - 2x^2 \, dx$

$= \left[3x^2 - \frac{2x^3}{3}\right]_0^3$

$= (27 - 18) - (0 - 0)$

$= \underline{\underline{9 \text{ u}^2}}$ [3]

(d) $\int_1^3 g(x)\, dx$

$\doteqdot \frac{h}{3}\left[y_F + y_L + 4(y_1 + y_3) + 2y_2\right]$

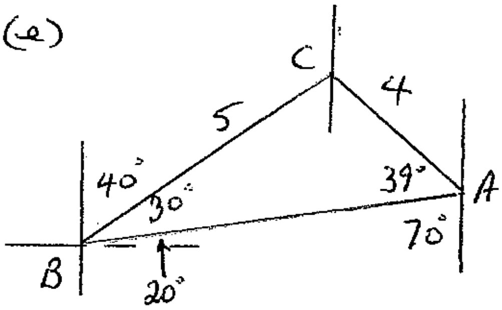
$= \frac{0.5}{3}\left[12 + 5 + 4(8+3) + 2(1.5)\right]$

$= \frac{1}{6}\left[17 + 44 + 3\right]$

$= \frac{1}{6} \times 64$ [2]

$= \underline{\underline{10\frac{2}{3}}}$

(e)



(i) $\underline{\angle CBA = 30°}$ let $\theta = \angle CAB$

$\frac{\sin\theta}{5} = \frac{\sin 30°}{4}$

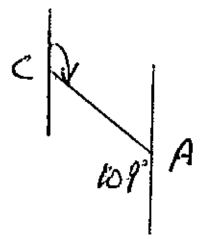
$\sin\theta = \frac{\sin 30° \times 5}{4}$

$= 0.625$

$\underline{\underline{\theta = 39°}}$ (nearest degree)

[3]

(ii)



Bearing of A from C $= 70° + 39°$

$= \underline{\underline{109°}}$ [1]

Question 9

(a) $y = 2x^2 - 2$

$y + 2 = 2x^2$

$x^2 = \frac{1}{2}(y+2)$

$$V = \pi \int_0^6 \frac{1}{2}(y+2)\,dy$$

$$= \frac{\pi}{2}\left[\frac{y^2}{2} + 2y\right]_0^6$$

$$= \frac{\pi}{2}\left[(18+12) - (0+0)\right]$$

$$= 15\pi\ u^3 \qquad [3]$$

(b) (i) $P(WWW) = 0.8 \times 0.8 \times 0.6$

$= 0.384$ [1]

(ii) P(at least 1 Beastie win)

$= 1 - P(\text{no Beastie wins})$

$= 1 - 0.2 \times 0.2$

$= 0.96$ [2]

(c) (i) $A_6 = 20000 - 6M$ [1]

(ii) $A_7 = (20000 - 6M)1.01 - M$

$A_8 = \left[(20000-6M)(1.01) - M\right]1.01 - M$

$= (20000-6M)(1.01)^2 - M(1.01) - M$

$= (20000-6M)(1.01)^2 - M(1+1.01)$

[1]

(iii) By above

$A_n = (20000-6M)1.01^{n-6} - M(1 + 1.01 + 1.01^2 + \dots + 1.01^{n-7})$

$A_{36} = (20000-6M)1.01^{30} - M(1 + 1.01 + 1.01^2 + \dots + 1.01^{29})$

[2]

(iv) $A_{36} = 0$

$$(20000 - 6M)1.01^{30} - M\left(\frac{1(1.01^{30} - 1)}{1.01 - 1}\right) = 0$$

$$(20000 - 6M)1.01^{30} = M(34.785)$$

$$20000 - 6M = \frac{M(34.785)}{1.01^{30}}$$

$$20000 = 6M + \frac{M(34.785)}{1.01^{30}}$$

$$20000 = M\left(6 + \frac{34.785}{1.01^{30}}\right)$$

$$M = \frac{20000}{\left(6 + \frac{34.785}{1.01^{30}}\right)}$$

$= \$629$ (~~nearest~~ \$)

[2]

Question 10

(a) (i) $V = 2 - \sqrt{3}\cos t - \sin t$

$$\frac{dV}{dt} = \sqrt{3}\sin t - \cos t$$

[1]

(ii) when $t=0$, $\frac{dV}{dt} = \sqrt{3}\sin 0 - \cos 0$

$= -1$

$\therefore$ initially emptying [1]

(iii) Full when $\frac{dV}{dt} = 0$

$\sqrt{3}\sin t - \cos t = 0$

$\sqrt{3}\sin t = \cos t$

$\frac{\sin t}{\cos t} = \frac{1}{\sqrt{3}}$

$\tan t = \frac{1}{\sqrt{3}}$

$\therefore \quad t = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \dots$

Initially emptying $\therefore$ empty when $t = \frac{\pi}{6}$

$\therefore$ first full when $t = \frac{7\pi}{6}$

when $t = \frac{7\pi}{6}$ $\quad V = 2 - \sqrt{3}\cos\frac{7\pi}{6} - \sin\frac{7\pi}{6}$

$= 2 - \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2}\right)$

$= 2 + \frac{3}{2} + \frac{1}{2}$

$= 4\ m^3$

[3]

(b) (i) By similar Δ's, $\frac{x}{x+h} = \frac{2}{2q}$

$xq = x + h$

$xq - x = h$

$x(q-1) = h$

$x = \frac{h}{q-1}$

[2]

(ii) $V = \frac{1}{3}\pi(2q)^2(x+h) - \frac{1}{3}\pi(2)^2 x$

$= \frac{1}{3}\pi(2q)^2\left(\frac{h}{q-1} + h\right) - \frac{1}{3}\pi(2)^2\left(\frac{h}{q-1}\right)$

$= \frac{1}{3}\pi(2q)^2\left(\frac{hq}{q-1}\right) - \frac{1}{3}\pi(2)^2\left(\frac{h}{q-1}\right)$

$= \frac{1}{3}\pi(2)^2\left(\frac{h}{q-1}\right)(q^3-1)$

$= \frac{4}{3}\pi\left(\frac{h}{q-1}\right)(q-1)(q^2+q+1)$

$= \left(\frac{4\pi h}{3}\right)(q^2+q+1)$ [2]

(iii) $2 + h + 2q = 12$

$h = 10 - 2q$

$V = \frac{4\pi h}{3}(q^2+q+1)$

$= \frac{4\pi}{3}(10-2q)(q^2+q+1)$

$= \frac{4\pi}{3}(8q^2 + 8q - 2q^3 + 10)$

$\frac{dV}{dq} = 0$ for max Vol.

$\frac{4\pi}{3}(16q + 8 - 6q^2) = 0$

$q = \frac{-16 \pm \sqrt{16^2 - 4(-6)(8)}}{2(-6)}$

$q = \frac{-16 \pm \sqrt{448}}{-12}$

$q = 3.10 \quad (q > 0)$

Check for max

$\frac{d^2V}{dq^2} = \frac{4\pi}{3}(16 - 12q)$

when $q = 3.10$

$\frac{d^2v}{dt^2} = -88.7$

(< 0)

$\therefore$ Max Volume when $q = 3.10$

[3]